These are complete elliptic integrals of the second and first kinds respectively, and can be readily evaluated. See Peirce's *Short Table of Integrals*, pp. 118–119, and Bromwich's *Infinite Series*, p. 162, ex. 6.

Using the notation employed in the former, we have

$$u = \tfrac{1}{3}\{12\pi\sqrt{2}E(\sqrt{\tfrac{1}{2}},\ \phi) - 16\pi\sqrt{2}F(\sqrt{\tfrac{1}{2}},\ \phi)\}.$$

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

188. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Solve the congruence $3x^2 + 4x + 5 \equiv 0$ [mod 20].

SOLUTION BY WALTER C. EELLS, Whitworth College.

By inspection, solutions of the two congruences,

$$3x^2 + 4x + 5 \equiv 0 \pmod{4}, \tag{1}$$

$$3x^2 + 4x + 5 \equiv 0 \pmod{5}, \tag{2}$$

are $x = \pm 1$ for (1), and $x = 0, 2$ for (2).

Then we have the four systems of linear congruences,

$$(3)\ \begin{cases} x \equiv 1 \pmod{4}, \\ x \equiv 0 \pmod{5}, \end{cases} \qquad (4)\ \begin{cases} x \equiv 1 \pmod{4}, \\ x \equiv 2 \pmod{5}, \end{cases}$$

$$(5)\ \begin{cases} x \equiv -1 \pmod{4}, \\ x \equiv 0 \pmod{5}, \end{cases} \qquad (6)\ \begin{cases} x \equiv -1 \pmod{4}, \\ x \equiv 2 \pmod{5}. \end{cases}$$

To solve system (3), substitute $x = 1 + 4y$ from first congruence in second congruence,

$$1 + 4y \equiv 0 \pmod{5};$$

whence

$$y = 1 + 5k,$$

and

$$x = 1 + 4y = 1 + 4(1 + 5k) = 5 + 20k.$$

Hence, $x \equiv 5 \pmod{20}$, yielding one solution of given congruence. Similarly, systems (4), (5), (6) yield $x = 17, 15, 7$, giving altogether four independent roots of the congruence $3x^2 + 4x + 5 \equiv 0 \pmod{20}$.

Also solved by H. C. FEEMSTER, LOUIS CLARK, S. LEFSCHETZ, E. B. ESCOTT.

No solution of 189 has been received.